

Name:

Solutions

Directions: In order to receive full credit, you must show all relevant work. Give all irrational solutions in simplified radical form.

1. Factor $x^2 + 6x - 27$

$$(x+9)(x-3)$$

2. Solve $x^2 + 36 = 0$ for all solutions (real and imaginary)

$$x^2 = -36 \quad \sqrt{x^2} = \pm \sqrt{-36} \quad x = \pm i\sqrt{36} \Rightarrow x = \pm 6i \\ \{6i, -6i\}$$

3. Solve $6m^2 - 24m = 0$ for all solutions (real and imaginary)

$$6m(m-4) = 0 \\ 6m = 0 \quad m-4 = 0 \\ m = 0 \quad m = 4$$

$$\{0, 4\}$$

4. Solve $3y^3 - 24y = 0$ for all solutions (real and imaginary)

$$3y(y^2 - 8) = 0 \\ 3y = 0 \quad y^2 - 8 = 0 \quad y^2 = 8 \quad y = \pm \sqrt{8} \quad y = \pm 2\sqrt{2} \\ y = 0 \quad \{0, 2\sqrt{2}, -2\sqrt{2}\}$$

5. Factor $9x^2 - 64$

$$(3x-8)(3x+8)$$

6. Solve $3x^2 - 5x - 28 = 0$ for all solutions (real and imaginary)

$$(3x+7)(x-4) = 0 \\ 3x+7 = 0 \quad x-4 = 0 \\ x = -\frac{7}{3} \quad x = 4$$

$$\{-\frac{7}{3}, 4\}$$

7. Factor $y^3 - 8k^3$

$$(y-2k)(y^2 + 2ky + 4k^2)$$

8. Factor $2y^3 - 12y^2 - 32y$

$$2y(y^2 - 6y - 16) = 2y(y - 8)(y + 2)$$

9. Solve $x^4 + 2x^2 - 63 = 0$ for all solutions (real and imaginary)

$$(x^2 + 9)(x^2 - 7) = 0$$

$$\begin{array}{l} x^2 + 9 = 0 \quad x^2 = -9 \quad x = \pm \sqrt{-9} \quad x = \pm 3i \\ x^2 - 7 = 0 \quad x^2 = 7 \quad x = \pm \sqrt{7} \end{array} \quad \{3i, -3i, \sqrt{7}, -\sqrt{7}\}$$

10. Solve $x^3 + 3x^2 - 18x - 40 = 0$ for all solutions (real and imaginary) (Hint: $x + 2$ is one factor of the polynomial)

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -18 & -40 \\ & & -2 & -2 & 40 \\ \hline & 1 & 1 & -20 & 0 = \text{Remainder} \end{array}$$

$$x^2 + x - 20 = 0 \quad (x + 5)(x - 4) = 0$$

$$\{-2, -5, 4\}$$

11. Solve $x^3 - 7x^2 - 8x + 56 = 0$ for all solutions (real and imaginary)

$$x^2(x - 7) - 8(x - 7) = 0$$

$$(x - 7)(x^2 - 8) = 0$$

$$\begin{array}{l} x - 7 = 0 \quad x = 7 \\ x^2 - 8 = 0 \quad x^2 = 8 \\ x^2 = \pm \sqrt{8} \quad x = \pm 2\sqrt{2} \end{array} \quad \{7, 2\sqrt{2}, -2\sqrt{2}\}$$

12. A polynomial function of degree 3 has the solution set $\{0, -3, 7\}$. Determine an equation of the function in standard form.

$$\begin{array}{l} x = 0 \quad x = -3 \quad x = 7 \\ x + 3 = 0 \quad x - 7 = 0 \\ x(x + 3)(x - 7) = 0 \quad x(x^2 - 4x - 21) = 0 \quad x^3 - 4x^2 - 21x = 0 \end{array}$$

13. A polynomial function of degree 3 has the solution set $\left\{-4, -\frac{2}{3}\right\}$ given that -4 is a double root. Determine an equation of the function in standard form.

$$\begin{array}{l} x = -4 \quad x = -4 \quad x = -\frac{2}{3} \\ x + 4 = 0 \quad x + 4 = 0 \quad 3x = -2 \\ 3x + 2 = 0 \\ (x + 4)(x + 4)(3x + 2) = 0 \\ (x^2 + 8x + 16)(3x + 2) = 0 \\ 3x^3 + 26x^2 + 64x + 32 = 0 \end{array}$$

14. Write an equation in standard form for the degree 2 polynomial function with the given solution set $\{\sqrt{11}, -\sqrt{11}\}$ $x = \sqrt{11}$ $x = -\sqrt{11}$

$$(x - \sqrt{11})(x + \sqrt{11}) = 0$$

$$x^2 + \sqrt{11}x - \sqrt{11}x - \sqrt{121} = 0$$

$$x^2 - 11 = 0$$

15. Write an equation in standard form for the degree 3 polynomial function with the given solution set $\{0, 4i, -4i\}$

$$x = 0 \quad x = 4i \quad x = -4i \quad x(x^2 + 16) = 0$$

$$x(x - 4i)(x + 4i) = 0 \quad x^3 + 16x = 0$$

16. Simplify $\frac{8y}{7} - \frac{y+28}{7} = \frac{8y - y - 28}{7} = \frac{7y - 28}{7} = y - 4$

17. Simplify: $\frac{3 \cdot (w+4)}{3 \cdot 24} + \frac{(w-1)4}{(18)4}$ LCD: 72

$$\frac{3w+12}{72} + \frac{4w-4}{72} = \frac{3w+12+4w-4}{72} = \frac{7w+8}{12}$$

18. Simplify $\frac{5}{x} - \frac{3}{x+2} = \frac{(x+2)5}{(x+2)x} - \frac{x \cdot 3}{x \cdot (x+2)} = \frac{5x+10}{x(x+2)} - \frac{3x}{x(x+2)}$

$$\frac{5x+10-3x}{x^2+2x} = \frac{2x+10}{x^2+2x}$$

19. Simplify: $\frac{x^2-2}{x^2} + \frac{3-2x}{6x} = \frac{6 \cdot (x^2-2)}{6 \cdot x^2} + \frac{x \cdot (3-2x)}{x \cdot 6x} = \frac{6x^2-12}{6x^2} + \frac{3x-2x^2}{6x^2}$

$$= \frac{6x^2-12+3x-2x^2}{6x^2} = \frac{4x^2+3x-12}{6x^2}$$

20. Simplify $\frac{2}{7x^2y} + \frac{3}{2xy^2} = \frac{24 \cdot 2}{24 \cdot 7x^2y} + \frac{7x \cdot 3}{7x \cdot 2xy^2} = \frac{4y}{14x^2y^2} + \frac{21x}{14x^2y^2}$

$$\frac{4y+21x}{14x^2y^2}$$

21. Solve $\frac{4}{6} = \frac{2w}{21}$

$$\frac{4}{6} = \frac{2y}{21}$$

$$21 \cdot \frac{2}{3} = \frac{2y}{21} \cdot 21 \quad 7 \cdot 2 = 2y$$

$$7 = y$$

$$\{7\}$$

22. Solve $\frac{6}{x+2} = \frac{9}{x+3}$

$$\frac{6}{x+2} = \frac{9}{x+3} \quad \frac{6(x+3)}{x+2} = \frac{9(x+2)}{x+3}$$

$$(x+3) \cdot 6 = (x+2) \cdot 9$$

$$6x + 18 = 9x + 18$$

$$6x = 9x$$

$$0 = 3x$$

$$0 = x$$

$$\{0\}$$

23. Solve $\frac{9}{10} - y = \frac{4y}{5}$

$$10 \cdot \frac{9}{10} - 10 \cdot y = 10 \cdot \frac{4y}{5}$$

$$9 - 10y = 2 \cdot 4y$$

$$9 - 10y = 8y$$

$$9 = 18y$$

$$\frac{1}{2} = y$$

$$\left\{\frac{1}{2}\right\}$$

24. Solve $\frac{21}{h^2} - 1 = \frac{4}{h}$

$$h^2 \frac{21}{h^2} - h^2 \cdot 1 = h^2 \cdot \frac{4}{h}$$

$$21 - h^2 = 4h$$

$$0 = h^2 + 4h - 21$$

$$0 = (h+7)(h-3)$$

$$\begin{array}{l} h+7=0 \quad h-3=0 \\ h=-7 \quad h=3 \end{array}$$

$$\{-7, 3\}$$

25. Solve $1 + \frac{2}{y} = \frac{4}{y-3}$

$$y(y-3) \cdot 1 + y(y-3) \cdot \frac{2}{y} = y(y-3) \cdot \frac{4}{y-3}$$

$$y^2 - 3y + 2y - 6 = 4y$$

$$y^2 - y - 6 = 4y$$

$$y^2 - 5y - 6 = 0$$

$$(y-6)(y+1) = 0$$

$$\{6, -1\}$$

26. Solve $\frac{2}{x+1} - \frac{1}{x} = \frac{1}{6}$

$$6x(x+1) \cdot \frac{2}{x+1} - 6x(x+1) \cdot \frac{1}{x} = 6x(x+1) \cdot \frac{1}{6}$$

$$6x \cdot 2 - 6(x+1) = x(x+1) \quad 12x - 6x - 6 = x^2 + x$$

$$6x - 6 = x^2 + x$$

$$0 = x^2 - 5x + 6$$

$$0 = (x-2)(x-3)$$

$$\{2, 3\}$$